Mode-I fracture energy influence on the behavior of plain concrete beam

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Abstract: The principle aim of this research is concentrated to analyze the effect of cracks and their propagations on the mechanical behavior of a quasi-brittle material such as concrete. The singularity (stress concentration to infinity at the tip of crack) is avoided by using the principal of fracture energy with the fictitious crack approach. The concrete crack is divided into two major zones; the first one is the fracture zone (a combination of bridging effect and the cohesive microscopic cracking) which obeys a special law permitting the transmission of stress across the two faces of crack, this zone is considered as partially cracked concrete. When the opening of the crack exceeds a specific value, this zone is converted to a real crack (an open crack) and cannot transmit any stress across the two faces of a crack. The program of finite element used in this research is prepared by the researcher using discrete-crack approach with the experimental data obtained from the flexural test on notched beam loaded under three-point bending, where fracture mode I is dominated. The response of the applied load-crack mouth opening displacement (CMOD) with appropriate fracture energy is selected. The results show that the cohesive microscopic cracking zone for the plain concrete is very wide. The cohesive stress distributions across the microcracks with the corresponding crack openings are drawn from the first crack appearance till the beam failure.

تأثير طاقة التصدع طور I على سلوكية العتبة الخرسانية غير المسلحة

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الخلاصة

يتركز الهدف الأساسي من هذا البحث على تحليل تأثير التصدعات وتطورها على السلوكية الميكانيكية للمواد الشبه هشة مثل الخرسانة. يتم تجنب ألأحادية (تركيز الأجهادات إلى مالا نهاية عند رأس التصدع) باستخدام مبدأ طاقة التصدع من خلال التمثيل الخيالي للتصدع، لذا فان تصدع الخرسانة ينقسم إلى جزئين رئيسين، الجزء الأول منطقة التصدع (مركبة من فعل التجسير والتصدعات المجهرية التماسكية) وتخضع إلى علاقة تسمح بانتقال الأجهادات عبر وجهي التصدع ويمكن اعتبارها منطقة خرسانة متصدعة جزئيا. عندما تتجاوز فتحة التصدع مقدار محدد، تتحول هذه المنطقة إلى صدع حقيقي غير قابل لنقل الإجهاد عبر وجهي الصدع. برنامج العناصر المحددة المستخدم في هذا البحث معد من قبل الباحث باستخدام اسلوب التمثيل- الحقيقي للتصدع مع نتائج متحصل عليها من تجربة ألانحناء على عتبة مثلومة عند المنتصف ومحملة عند ثلاث نقاط، حيث يسيطر التصدع طور I. تم اختيار طاقة التصدع المناسبة لاستجابة الحمل المطبق مع أتساع فتحة أسفل الثلمة. أظهرت النتائج بأن منطقة التصدع المجهرية التماسكية ذات أتساع كبير. رسمت توزيعات الإجهاد التماسكية عبر التصدعات المجهرية مع الفتحات العائدة لها منذ ظهور التصدع الأول إلى فشل العتبة.

1- Introduction

Concrete is conventionally assumed to behave in an elastic-brittle manner under tension. However, the validity of this hypothesis depends on the scale of the element and the type of material analyzed. In general, concrete behaves as a quasi-brittle material, which is heterogeneous due to the presence of different phases, interfaces, pores, flaws and other defects, even before it is loaded. The failure is, therefore, not brittle but gradual. These flaws, especially the microcracks grow in a stable way while the structure is being loaded and when they coalesce into fractures, they could cause the collapse of the structure.

Current structural design does not take into account the tensile strength, and is only based on elasticity and plasticity theories. It has been demonstrated that these conservative methods provide satisfactory results for design. However, most design equations are not based on physical principles but on empirical observations, and therefore need to be constantly calibrated for new materials and structural types.

Fracture mechanics provides a general failure theory on which design can be based. Linear elastic fracture mechanics is based on Griffith's theory of 1920 [1]. However, this cannot be applied directly to concrete because it is only valid for homogeneous elastic-brittle materials such as glass, and the behavior of concrete differs considerably. This is also the reason why fracture mechanics is absent from current codes of practice. In 1957 Irwin [2] proposed some modifications to Griffith's theory. These changes made the theory valid for elastic-plastic materials, such as metals with a limited ductility.

1.1- Linear elastic fracture mechanic

In this section, the concepts of the theory of linear elastic fracture mechanics (LEFM) are introduced, which are essential to understand the subsequent development of this theory.

Typically, we have three different modes to resolve the general state of stress, and they are Mode I, Mode II and Mode III respectively as shown in figure (1). Mode I refer to a planar symmetric state of stress, which causes a crack to open. In this case the crack faces are displaced normal to their plane. Mode II refers to a planar anti-symmetric state of stress, which causes a relative displacement of the crack faces in their own plane, and, finally, mode III refers to a state of stress that causes a relative displacement of the crack faces out of their plane. They are also called, opening mode, sliding mode and tearing mode respectively.

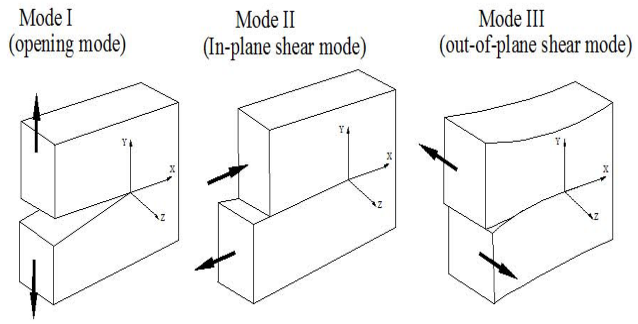


Figure (1): Three independent modes of deformation at the crack tip.

1.2- The brittle fracture theory by Griffith

It was in 1920 Griffith introduced the basis of LEFM. Before that, there was no explanation for the differences between the theory and the experimental observations in some aspects of the behavior of hard brittle materials. Griffith showed that because of the cracks in the material, the observed value of the tensile strength was lower than the expected value. At a hole or tip of a crack, an increase of the stress is expected due to the stress concentration. However, at points far away, the stress flow is not influenced by the hole.

1.3- The Irwin theory of brittle fracture

In the Griffith theory [1] of brittle fracture, the only points where the stress state is singular is in the cracks tips, while the rest of the body of the structure remains elastic. However, Irwin [2] showed that in plastic materials, there is no singularity but a plastic zone near the crack tip. Besides this fact, Irwin also realized that closer to the crack tips, the three stress components were the same without taking into account the shape of the body and how it is loaded.

2- Fracture mechanics to structural concrete.

The nonlinearity of concrete behavior can be illustrated by the following figure (2). In an idealized load-deflection curve corresponding to unaxial tension [3], four different stages can be distinguished; the first consists of a linear response, the second stage is nonlinear leading to the peak load. The third and forth stages are characterized by an increase in the deformation with a decrease in the stress. Such response is called strain softening in tension, or simply tension beam softening to distinguish it from the strain softening in compression. These kinds of materials are called quasi-brittle materials.

Recent studies on the fracture behavior of concrete reveal that some fracture characteristics differ from those normally observed in metallic materials. It is well known that microcracks exist in concrete even before it has been loaded. It is now clear that a fracture theory capable of include in it a description of the material softening taking place in the fracture process zone.

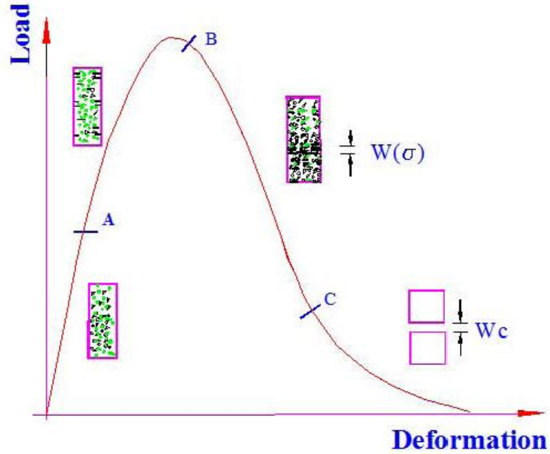
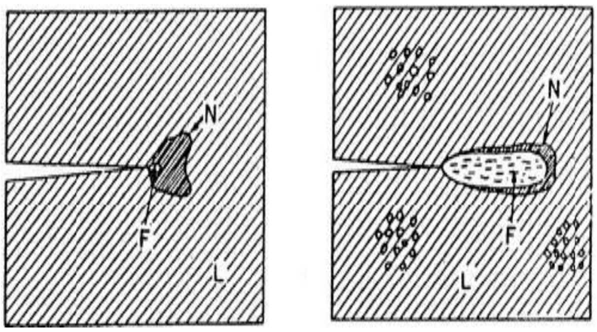


Figure (2): Typical load-deformation response of a quasi-brittle material in tension.

Such a theory will necessarily be nonlinear but one must distinguish the ductile-materials such as metals, from that applicable to quasi-brittle materials, such as rocks, concrete and ceramics. This is because in ductile materials the fracture process zone though small is surrounded by a large plastic zone, whereas in quasi-brittle material the fracture process zone practically occupies the entire zone of nonlinear deformation. In contrast, the nonlinear zone is absent in quasi-brittle materials. The above remarks are schematically illustrated in figure (3).



(a) (b)

Figure (3): Fracture plastic zone a- Ductile-brittle (metals) b- quasi-brittle (concrete) [4]

2.1- Fracture process zone and tension–softening phenomenon in concrete.

Concrete is a heterogeneous material that consists of aggregates and cement pastes bonded together at the interface, and the material is inherently weak in tension due to the limited bonding strength and various preexisting microcracks and flaws that form during hardening of the matrix. The tensile strength of concrete approximately ranges from 6 to 15 percent of its compressive strength. Under external loading, a tension zone forms near the crack tip, in which complicated microfailure mechanisms take place. These fracture processes include microcracking, crack deflection, crack branching, crack coalescence, and debonding of the aggregate from the matrix, which are examples of inelastic toughening mechanisms that coexist with a crack when it propagates. In concrete, the inelastic zone at the crack tip is extensively developed and therefore, in principle, LEFM cannot be used to study the fracture of concrete. Figure (4) schematically illustrates the formation of an inelastic zone in concrete, which is known as a fracture process zone (FPZ) that can be roughly divided into a bridging zone and a microcracking zone, along with two idealizations of the FPZ. It is known that bridging is a result of the weak interface between the aggregates and the cement pastes, and it is an important toughening mechanism in concrete. Within the damage zone the effective modulus of elasticity is reduced from that of the undamaged material E to E*, if the process zone is modeled as a region of strain softening as shown in figure (4 - b).

Hillerborg et al. (1976) [6] envisioned a fictitious crack method (FCM) in place of the physical FPZ and subjected it to closure tractions, as shown in figure (4-c). As is illustrated in the figure, the closure stress associated with the bridging grains and microcracks is a maximum at the tip of the FPZ and decreases to zero at the continuous crack tip where the crack opening reaches its critical value w_c, beyond which an open crack forms. Known as the tension-softening phenomenon, the relation between the closure stress and the crack opening with which the fracture energy of concrete is completely defined describes the local material behavior inside the FPZ when fracture takes place in concrete.

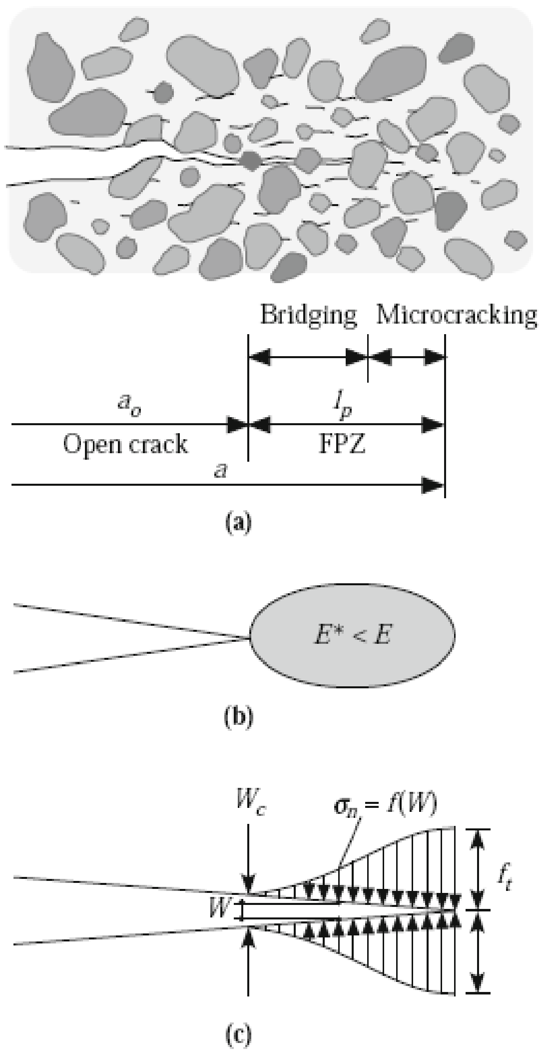


Figure (4): Concept of FPZ and tension-softening in concrete: (a) FPZ in front of an open crack, (b) reduced effective modulus of elasticity inside FPZ, and (c) tension-softening inside FPZ [5].

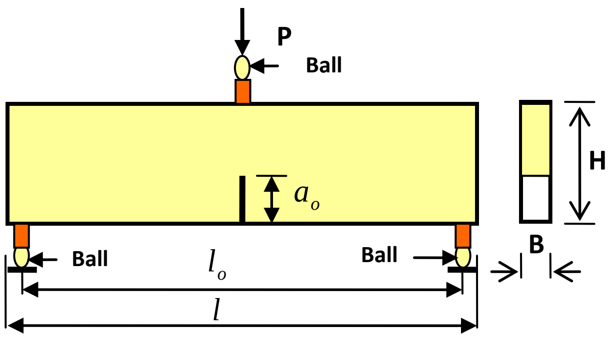
2.2- Fracture energy G_F and tension-softening relation in concrete.

As just discussed, fracture of concrete initiates in the FPZ ahead of an open crack through complicated micro-failure processes, and the fracture energy is consumed in overcoming the resistance of various toughening mechanisms to form an open crack at the end of the FPZ. The amount of fracture energy required to break a unit area of concrete is generally regarded as a material property (although it varies slightly with size) that determines the fracture behavior of the material through the fundamental relationship between the cohesive stress and the crack opening in the FPZ, which is known as the tension-softening law of concrete. Just like the constitutive relationship of a continuous material that stipulates the fundamental material behavior (whether it is elastic or inelastic), the tension-softening law with the fracture energy as its defining characteristic is the constitutive relationship for the material in the FPZ that describes the transitional material behavior from the continuous state to the discontinuous state, in other words, how the tensile stress decreases with the increasing discontinuity in the FPZ.

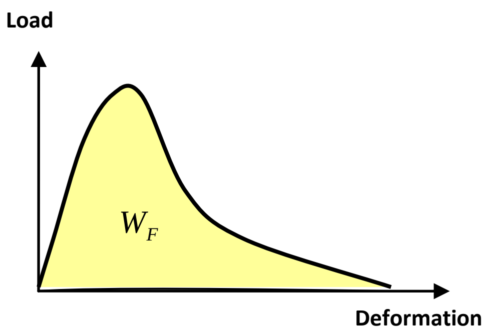
2.2.1- Fracture energy G_F .

The load-displacement relation is shown in figure (5), the area enclosed by the response curve and the horizontal axis represents the work done by the external load to fracture the beam. Suppose that the crack growth is stable and the work done by the external load is spent solely in crack propagation. Based on the Griffith energy criterion, crack growth in an elastic body in the equilibrium state is a natural process of energy transfer between the strain energy of the body and the fracture energy required for creating a new crack surface so that a state of minimum potential energy is achieved for the system at a given load level. In the present case, the work is consumed in breaking the un-notched part of the beam cross-section, the ligament ahead of the notch. Denoting the work of the external load by W_F and the ligament area by A_{Lig} , the energy needed to create a crack of unit area, G_F according to the RILEM Technical Committee 50-FMC (Fracture Mechanics of Concrete) 1991 [7], is obtained as:

$$G_F = \frac{W_F}{A_{Lig}} = \frac{W_F}{(H-a_0)B} \quad ---- (1)$$



(a)



(b)

Figure (5): Determination of fracture energy G_F based on the RILEM method: (a) notched beam under three-point bending, and (b) load-deformation relations.

Obviously, this relationship can also be obtained from the stress-crack opening relation at the notch tip, where an open crack has just been created by fracturing the intact material there. Notice that the area enclosed by this tension-softening curve with the horizontal axis as shown in figure (6), is exactly the fracture energy that is:

$$G_F = \int_0^{w_c} \sigma_n(w)\, dw \quad ....(2)$$

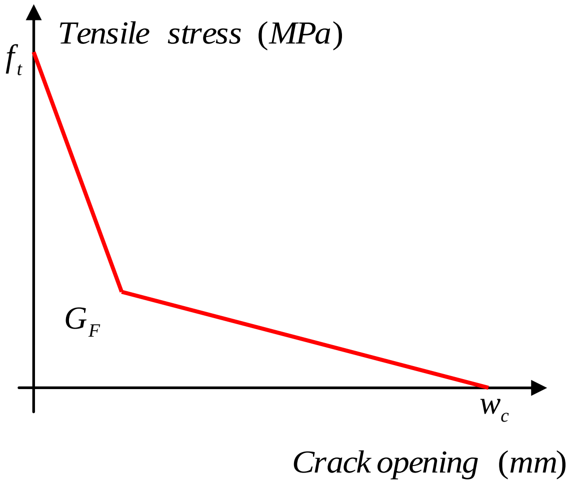


Figure (6): Tensile stress-crack opening relation.

2.2.2- Tension-Softening relation

As shown in figure (6), the tension-softening relation of concrete possesses two distinctive features: the steep descending slope caused by the rapid loss of tensile strength in the initial stage of softening and a long tail with the increasing crack-opening displacement, illustrating the persistent stress-transferring capability of aggregate interlocking in the FPZ. Three shapes of post-peak constitutive laws are represented below: the Linear of Hillerborg et al. [6] (1976), the exponential of Jawad M. [8] (1989) and the bilinear of CEB-FIP Model Code [9] (1993). These relationships are implemented and depicted in figure (7).

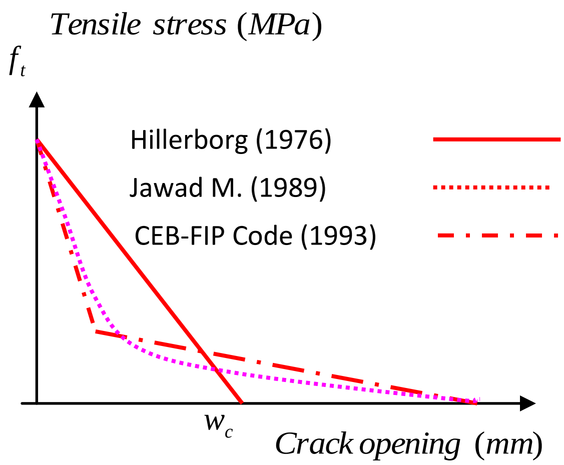


Figure (7): Proposed tension-softening models (a) by Hillerborg (1981) [6], (b) by Jawad M. (1989) [8] and CEB-FIP Code (1993) [9].

The relationship of the tension-softening used in this research [8] takes the form:

$$\sigma = -(f_t + \sigma_o)\, e^{\frac{-f_t w}{G_F + \sigma_o w_c}} + \sigma_0 \quad ....(3)$$

where

$$w_c = \frac{\ln\left(\frac{\sigma_o}{f_t + \sigma_o}\right) G_F}{f_t + \sigma_o \ln\left(\frac{\sigma_o}{f_t + \sigma_o}\right)} \quad ....(4)$$

σ_o = adjustable constant; w_c = maximum opening displacement. The pure exponential form with $w_c = \infty$ to linear form with $w_c = 2G_F / f_t$ is obtained by varying the constant value σ_o in the previous relation from 0 to a large number respectively. Figure (8), shows different shapes of tension-softening for concrete tensile strength $f_t = 3.3\,MPa$ and $G_F = 126$ N/m.

The model program adopts a criterion states: when the concrete is cracked, there is a sudden loss in the tensile stress at a crack tip due to the immediate energy liberation. Taking this into account, the tensile stress at the tip of cohesive crack initiation equal to ($\alpha\ f_t$), where α = (0.95-1.0).

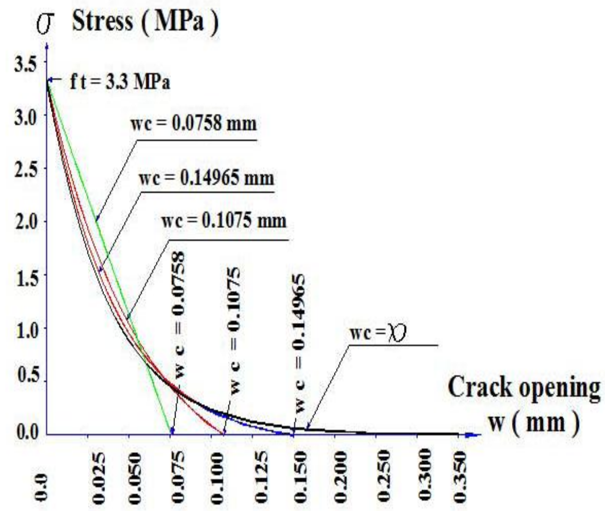


Figure (8): Different curve shapes of softening-tension for concrete $f_t = 3.3 MPa$ and $G_F = 126\, N/m$.

3- Applications and results

Bending test of a notched beam mode I fracture test.

The finite element program used in this application is prepared by the researcher to simulate the nonlinear behavior of plain and reinforced two-dimensional concrete beams by using the discrete cracking approach [8], [10]. This application is presented to gain an insight into the behavior of plain concrete under tensile stress with softening response. The experimental test is one of a series of testing carried out on three-point bend plain concrete specimens of similar geometry with different (notch/depth) ratios by Raghu Prasad B. K. et al. [11].

The dimensions, material properties and loading conditions of the test are shown in figure (9). The beam was modeled with mesh of (100) isoparametric quadrilateral of four nodes uncracked concrete elements, concentrating the small size elements at a tip of notch beam and on the expected direction of the crack propagation as shown in figure (10).

The load-crack mouth opening displacement (CMOD) response is shown in figure (11). There is a very good agreement between the experimental data and the results of the model with the selection of G_F equal to 343 N/m. The first microcrack appears at load equal to approximately 3.0 kN and the ascending branch of the response curve far away from that point is slightly beginning to incline as the crack is propagated. Before the pre-peak of the curve response at load nearly to 9.3 kN the microcrack is reached the elevation of 80 mm above a tip of beam notch and the stiffness of the beam is deteriorated, so that the inclination response curve becomes more flat and after that when the post-peak of the curve is exceeded the rate of response curve slop gradually dropped (load is decreased with crack opening displacement is increased) depending on the used value of fracture energy as it has a small value, the dropping rate is steeper till the divergence of numerical solution is occurred.

The influence of fracture energy has been studied varying the G_F between 0.0 (without fracture energy) to 450 N/m and keeping f_t= 40 MPa. As shown in figure (12), the curve of G_f= 343 N/m is more conform with the load- crack opening displacement response and this demonstrates that the fracture energy G_f is one of the important characteristics of the material and must be mentioned in the test as material properties. With G_f= 0.0, the first crack appearance depends mainly on the used concrete finite element size in the mesh. Considering beam notch as a real crack, the stress is concentrated at the tip of it causing the cracking of the concrete at this zone. The total stored fracture energy is immediately liberated; this is leading to steepest dropping of the load-crack opening displacement response of the beam immediately after cracking. As the value of G_F is increased, the released rate of fracture energy is gradually decreased leading to increase of maximum applied load and neglecting the effect of the used element size. When the fracture energy is increased to G_F = 140 N/m the maximum applied load is increased by approximately 150% to P_{max}= 7.5 kN. After that the increasing rate of maximum applied is rapidly reduced, for example when the fracture energy is increased from 140 N/m to 343 N/m (the fracture energy increased by 140%), the maximum applied load is approximately increased by 22% to 9.3 kN and from 343 N/m to 450 N/m (increased by 31.2%) the maximum applied load is increased by 4.3% to approximately 9.7 kN.

The figure (13) shows the crack propagation with the load at the moment of its appearance, due to the principal of fracture energy, the crack losses its ability of stress transmission gradually and correspondingly its stiffness. In order to propagate needs more external work (applied load) and so on till its strength capacity is reached and the crack propagation continues with decreasing applied load.

The following figures from (14) to (22) are showing the relationships between tensile stress of cohesive crack and cohesive crack opening at the load moment which is caused the subsequent appearance crack. At the beginning of loading, the gap (notch) represents a real crack so, the concentration of stresses at the tip of the gap is tended to be infinity (singularity) as a consequence the first crack appearance depends mainly on the size of used concrete element which means that the first crack appearance is needed zero load, if the size of used concrete element is tend to be zero due to the absence of the fracture energy G_f in the gap. With the size of concrete elements used in this test, the minimum load needed to start cracking of the beam is P= 2.91 kN, after that the fracture energy G_f introduces within the crack and is considered as a microscopic crack and begins to loss his ability to transmission of stress across the two faces of cracking gradually till the width of crack opening w exceeds the maximum permitted opening of the crack w_c, then the crack converts to a real crack and ceases of transmitting of stress through the two faces of cracking. This process eliminates the effect of concrete element size used in the model.

The crack is propagated towards of the applied load P. The crack openings width are increased due to the increasing of applied load, so that the cohesive stress within the crack is decreased to the values as shown in the previous figures. Figure (23) represents a numerically obtained load-deformation relation of a model of notched plain concrete beam under bending, and the growth of an FPZ at the notch tip based on the fictitious crack model by Hillerborg et al. [6]. The correspondence between the various points of the figure is meant to convey a clear picture on the FPZ and how it develops in the process of beam failure. As the figure shows, in the pre-peak region the tip stress at the notch reaches the tensile strength of concrete at point A, signaling the initiation of an FPZ. Upon reaching the peak load at point B, the FPZ has grown to a length of 80 mm, and the tensile stress at the notch tip decreases to 2.647 MPa as shown in figure (18). The tip stress of the notch drops to 0.565 MPa at point C as the FPZ stretches to length of 144 mm as shown in figure (22) and after that the divergence of numerical solution of the program is occurred and is never reached the point D, where the cohesive crack is converted to a real crack at maximum crack opening of 0.9257 mm in the post peak region, due to a good quality of concrete used in this test, which means high value of fracture of energy G_F and large maximum crack opening. As seen, the pre-peak nonlinearity and the tension-softening in the pre- and the early post-peak regions somewhere above point C are mainly the work of micro-cracking.

4- Conclusion

1- The using of the principle of fracture energy G_f is important to predict the tensile behavior of concrete and it is very necessary to insert this term in the researches, where the fracture energy G_f is considered as a restriction factor for crack propagation. While, if the fracture energy is neglected, the failure is occurred at the first appearance of cracking, if it is used in a plain concrete. Due to the above discussion, it is necessary to consider the fracture energy as a property of material.

2- The using of fracture energy principle avoids the concentration of stresses at tip of crack to infinity (singularity) but to a value equal to f_t, so the finite element mesh does not depend on the size of it, which was one of the defects facing the development of finite element method.

3- The strength of the beam is increased as the fracture energy G_F is increased but with decreasing rate and also the dropping rate of the descending softening curve is less.

4- The existence of cracks before applying the load has an influence on the strength of concrete. As the length of existence crack is increased, the strength of concrete will be decreased as well as failure load.

5- The representation of the microcrack is being considered as a part of crack and is governing by the transfer law of cohesive stress, which is decreased with increasing of crack opening.

6- As fracture energy value is increased, the maximum opening value w_c is increased, so that the cohesive crack able to transmit stresses for larger crack opening due to the bridging effect and is not converted to real crack in this application due to the divergence of numerical solution of the program. As one have seen in this research.

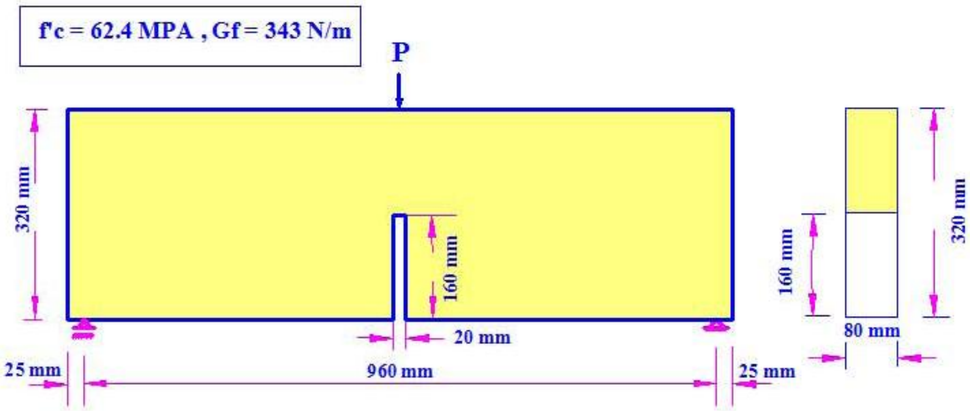


Figure (9): Test carried out by Raghu.

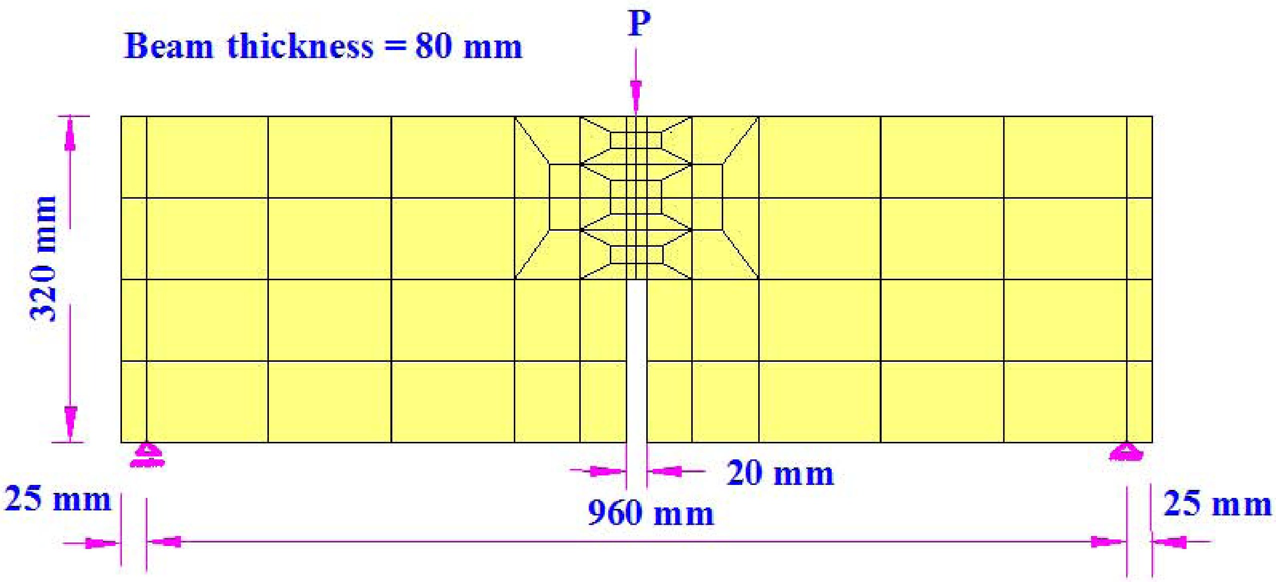


Figure (10): Finite element mesh of the beam.

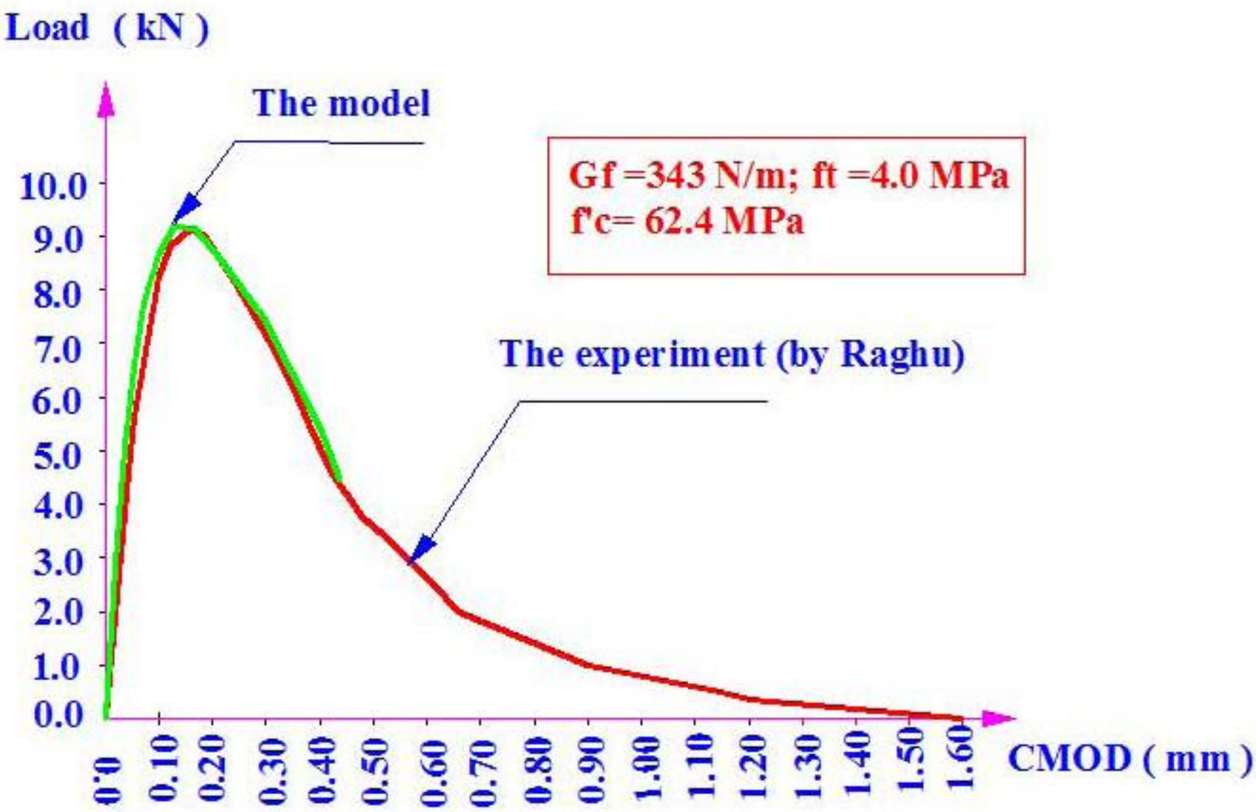


Figure (11): Fracture energy G_f influence on the Load-CMOD response.

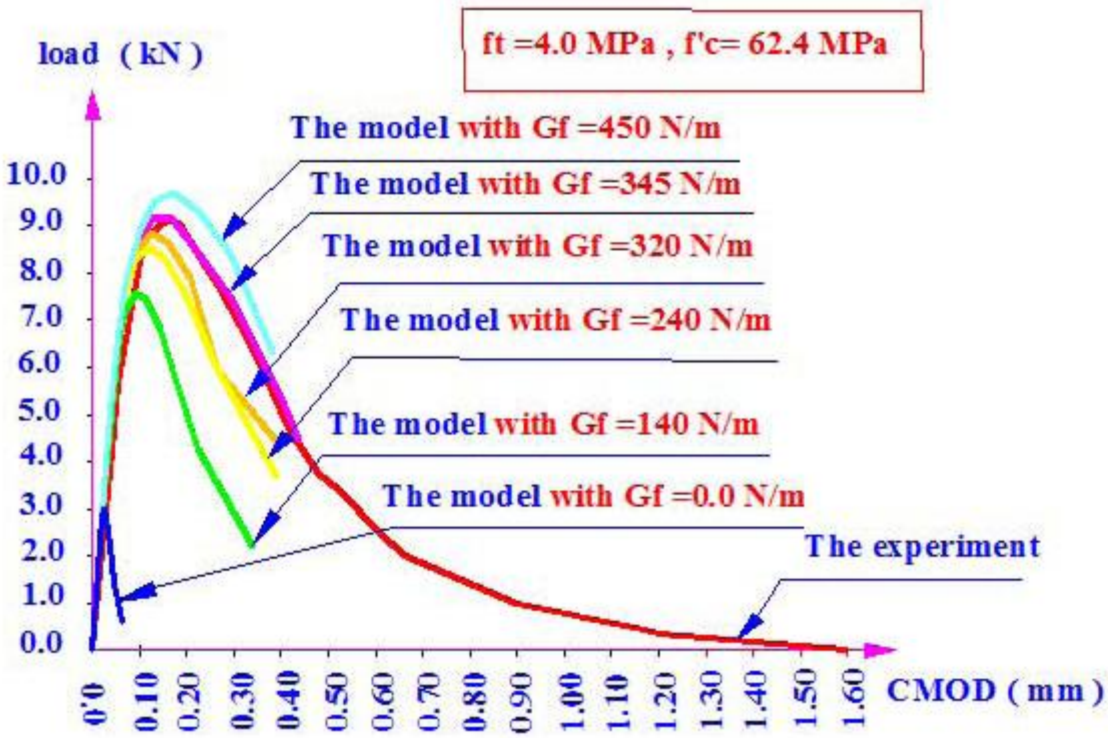


Figure (12): Fracture energy G_f influence on the Load-CMOD response.

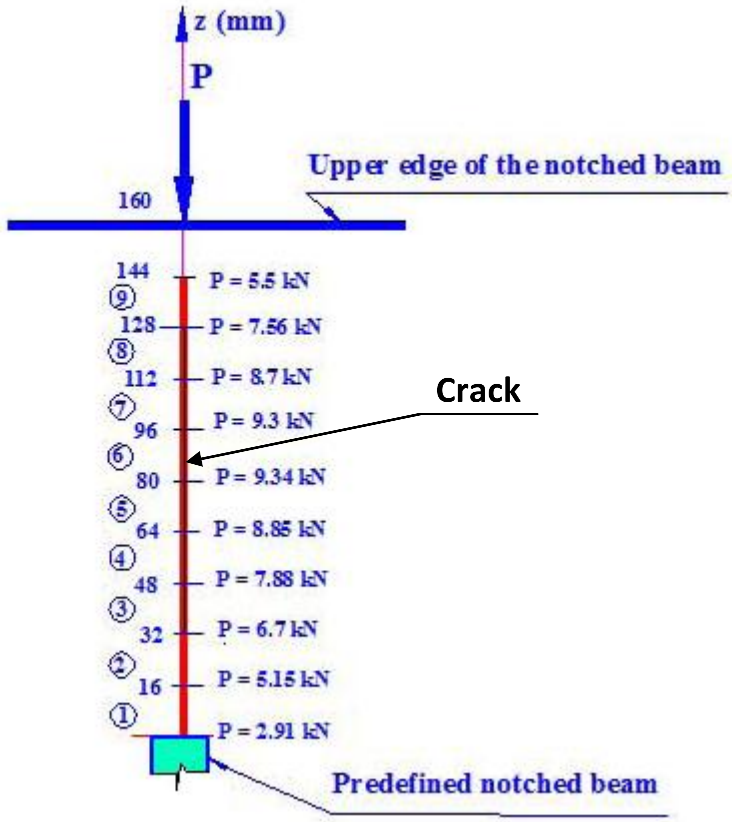


Figure (13): Load - Crack propagation on the middle of notched beam according to applied load.

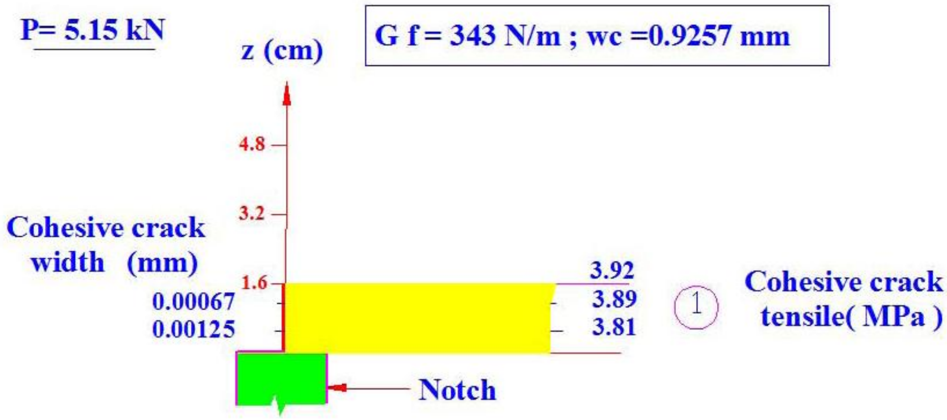


Figure (14): Crack cohesive stresses at load P= 5.15 kN

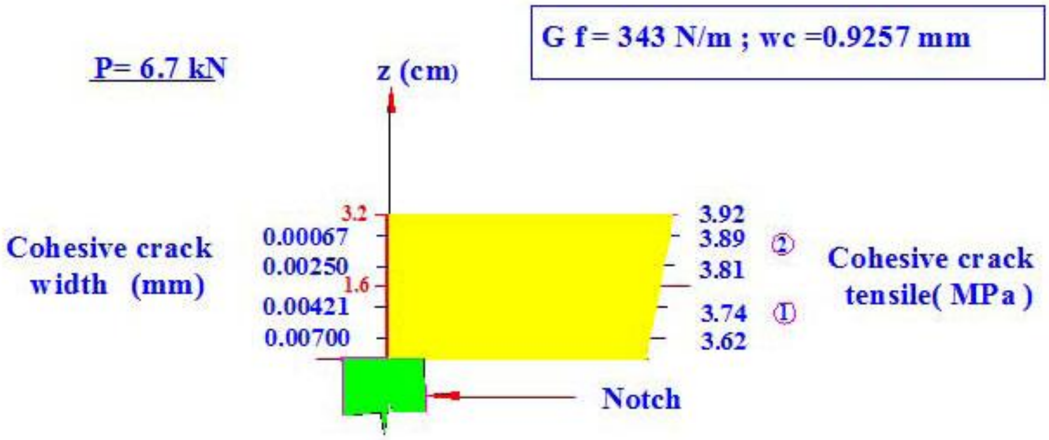


Figure (15): Crack cohesive stresses at load P= 6.7 kN

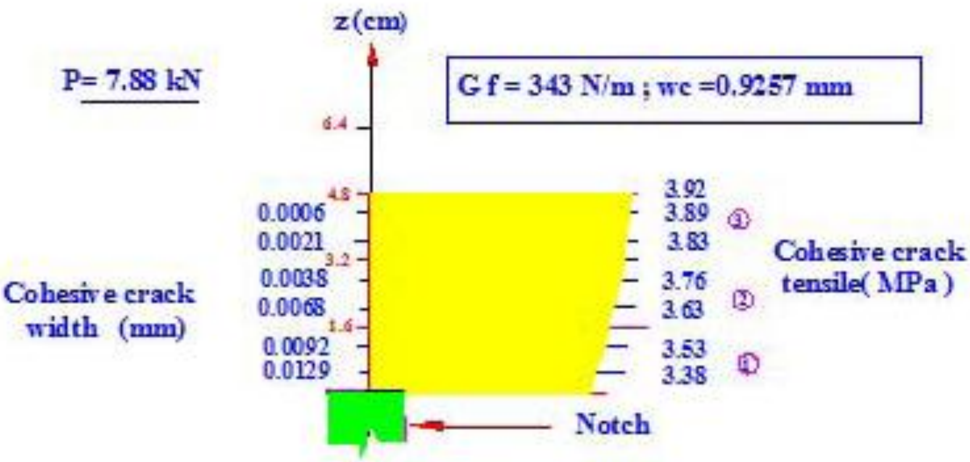


Figure (16): Crack cohesive stresses at load P= 7.88 kN

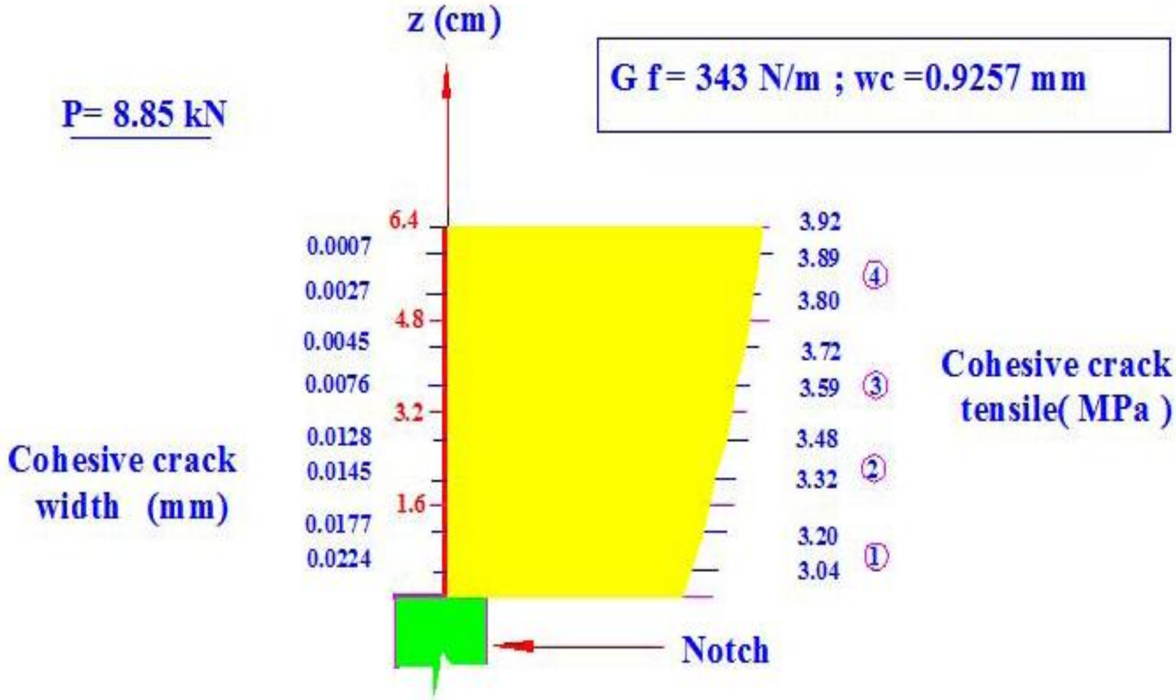


Figure (17): Crack cohesive stresses at load P= 8.85 kN

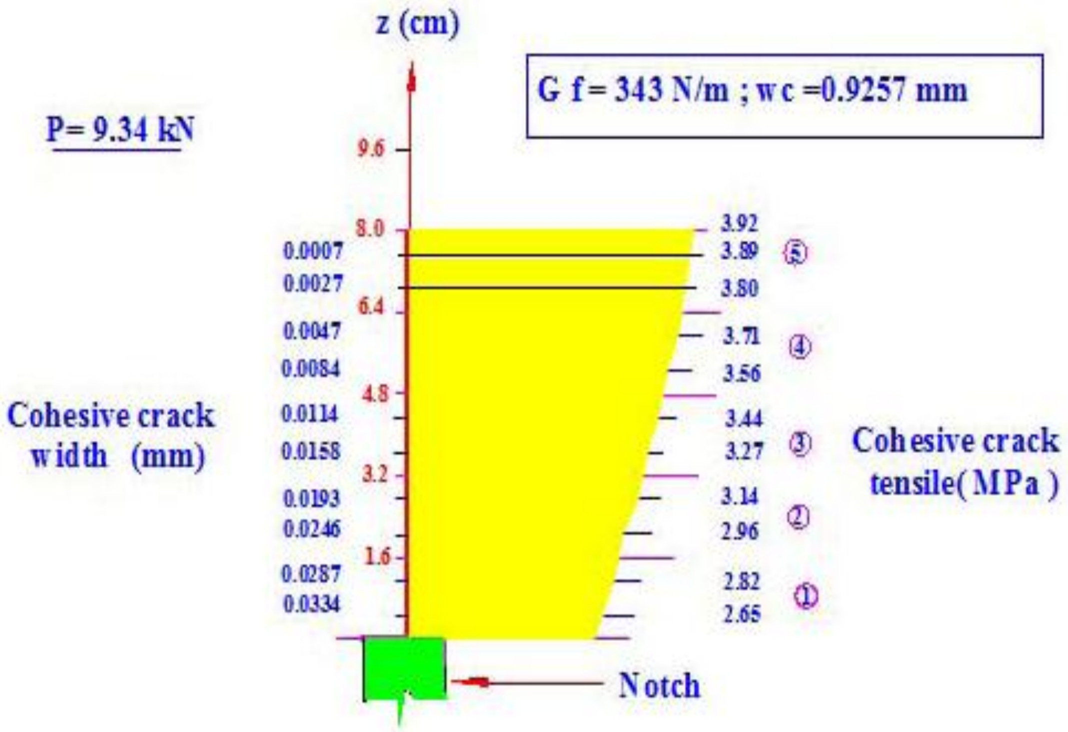


Figure (18): Crack cohesive stresses at load P= 9.34 kN

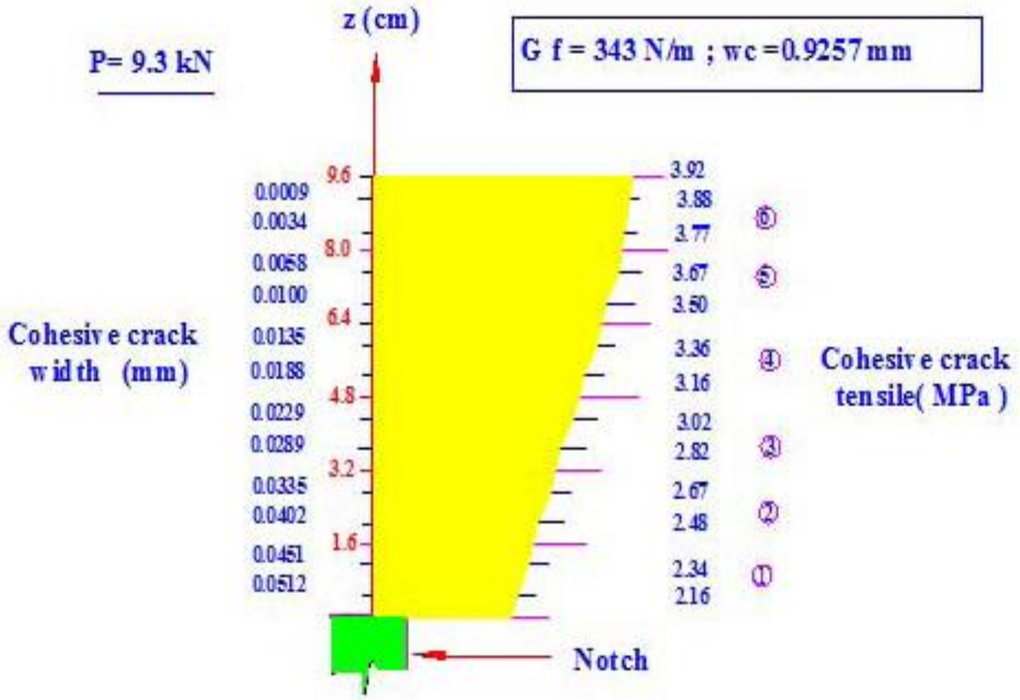


Figure (19): Crack cohesive stresses at load P= 9.3 kN

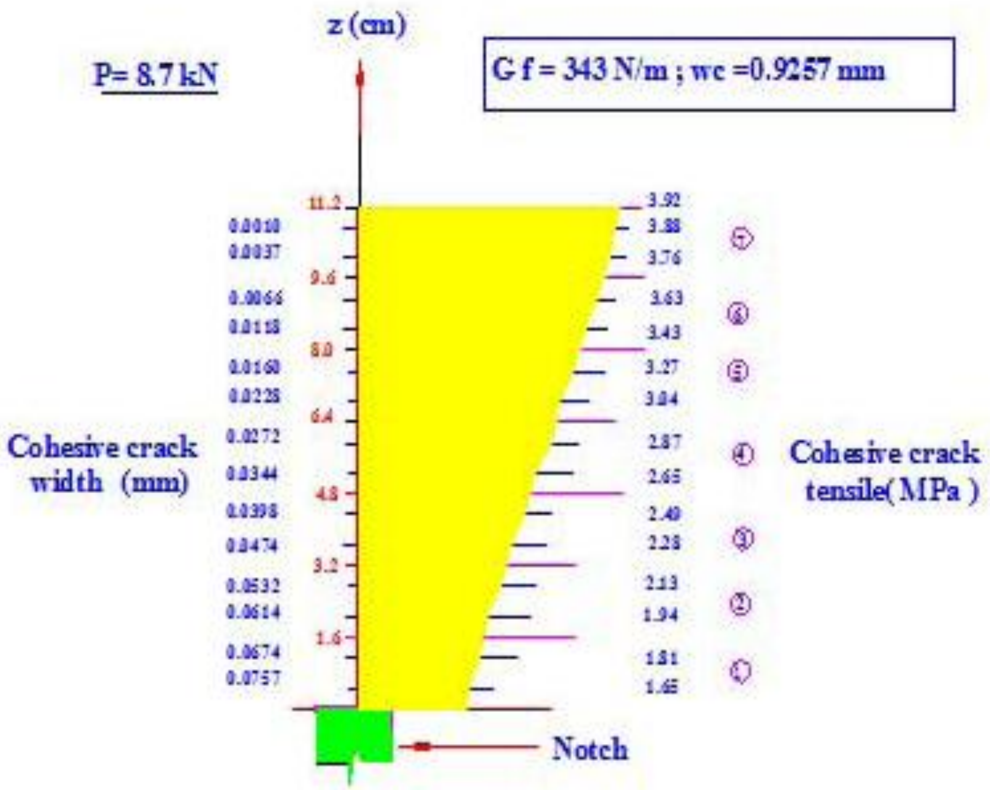


Figure (20): Crack cohesive stresses at load P= 8.7 kN

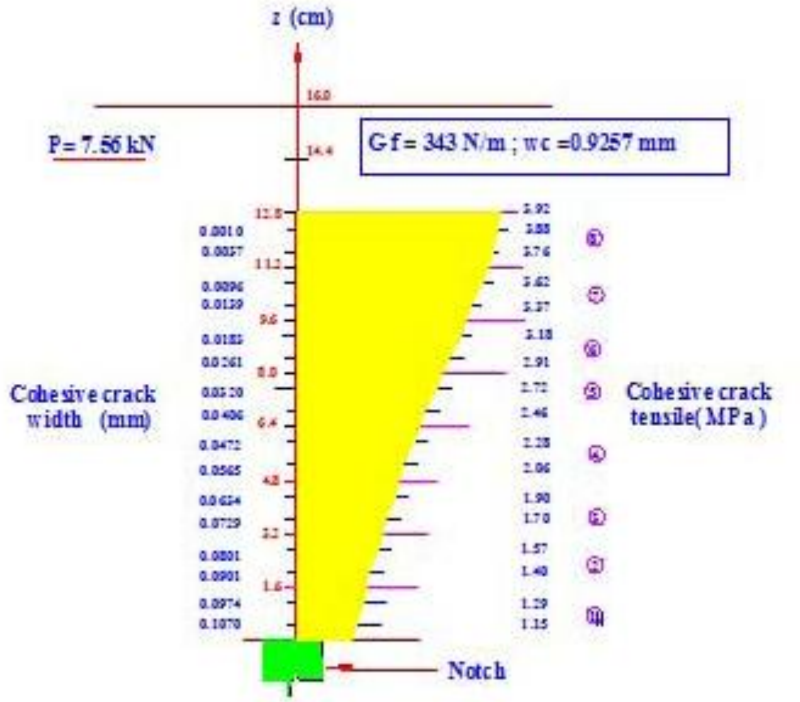


Figure (21): Crack cohesive stresses at load P= 7.56 kN

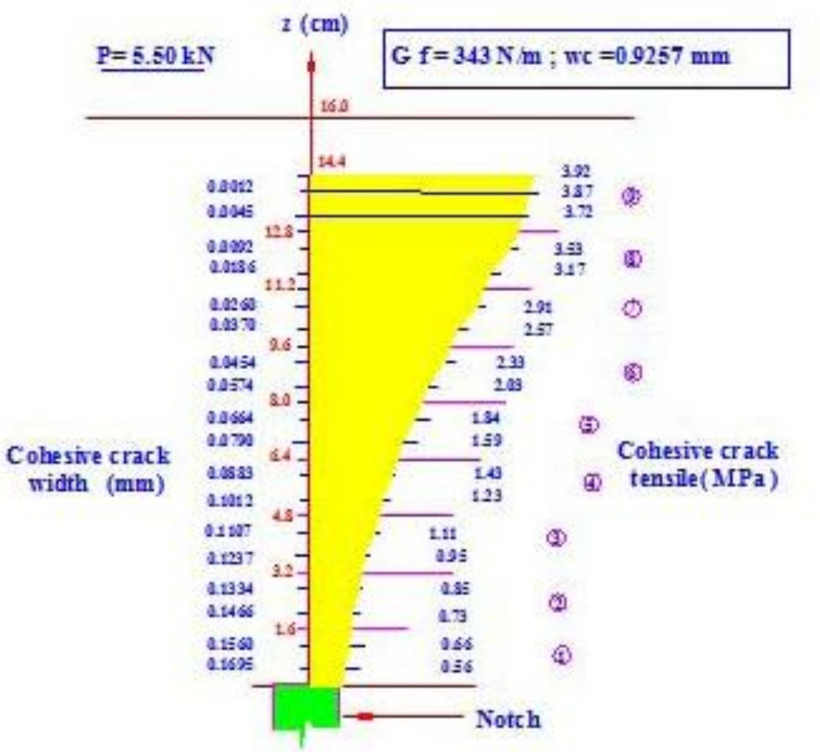


Figure (22): Crack cohesive stresses at load P= 5.5 kN

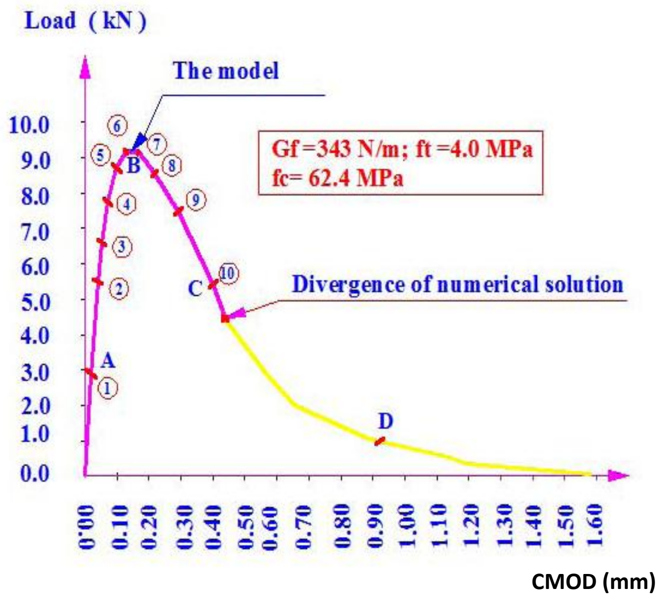


Figure (23): Crack cohesive appearance according to the Load- CMOD response curve.

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